SPUM 202 Lecture 7.

— Review

Quantum Dynamics.

[Schrödinger picture]

$|\psi(t)\rangle$

Time evolution

$$\hat{H}|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

$|\psi(t)\rangle$ is T.D.

Interaction picture.

$A(t)$, $|\tilde{\psi}(t)\rangle$.

[Heisenberg picture]

$\hat{A}(t)$

T. E.

Heisenberg EoM.

$$\underline{\frac{dA}{dt} = \frac{i}{\hbar}[H, A]}$$

operator $\hat{A}(t)$ T.D.

Time evolution operator. $U(t) = \exp\left(-\frac{iHt}{\hbar}\right)$

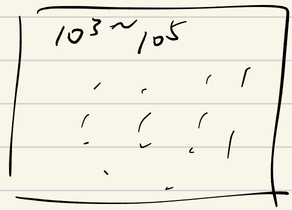
$$|\psi(t)\rangle = U(t)\ |\psi(t=0)\rangle. \qquad \text{S. P.}$$

$$\hat{A}^{(H)}(t) \equiv \underline{U^{\dagger}(t)\ \hat{A}^{(s)}\ U(t)} \quad — \quad (1.1)$$

$$\left\{ |\Psi\rangle_H \equiv |\Psi(t=t_0)\rangle_S \right. \quad —— (1.2)$$

Many-body system.

Quantum statistics.



Ψ . . .

$\hat{\rho}$: density matrix

$\hat{\rho} = |\Psi\rangle\langle\Psi| \longleftrightarrow$ pure state

$\hat{\rho} = \sum_{i=1}^{n} P_i |\Psi_i\rangle\langle\Psi_i| \longleftrightarrow$ mixed-state

Solve $\dfrac{d\hat{\rho}}{dt} = \dfrac{i}{\hbar}[H, \hat{\rho}]$ — (1,3)

- Schrödinger wave equation.
- Q harmonic oscillator.
- propagator, Feynman path integral.

MQM

Sakurai J.J.

— The First step in quantum field theory

Phonon. — quasi particle.

photon. : boson, Gauge boson. b^+, b. (Standard model)

$\begin{cases} \text{gauge boson}, \quad \text{spin}=1. \\ \text{scalar boson}, \quad \text{spin}=0. \end{cases}$

Force carrier.

① photon γ – EM.
② $W^\pm$, Z – weak
③ g : Strong.

H^0 Higgs boson

LHC

The theory of quantum field tells us

every boson corresponds to quantum fields.

$\gamma \longrightarrow$ E-/B- field. (EM wave)

$E_n = \hbar\omega(n + 1/2) \Leftrightarrow a^+, a$.

$\hat{n} = a^+ a$.

$E_0 = \hbar\omega \frac{1}{2}$

$\begin{cases} a^+ \to n+1. \\ a \to n-1. \end{cases}$

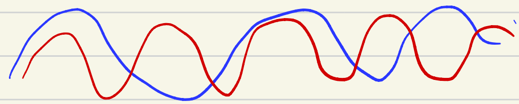
$b^+, b \leftarrow c^+ c^-$

— Quantization of EM field

single mode EM wave

ω, $\hat{\varepsilon}$

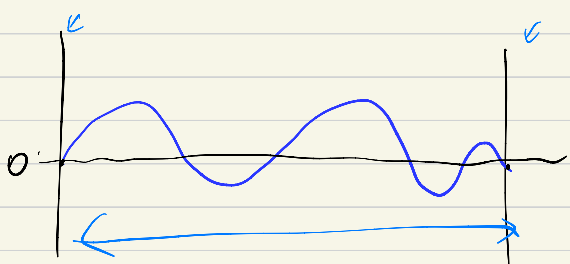
frequency, polarization

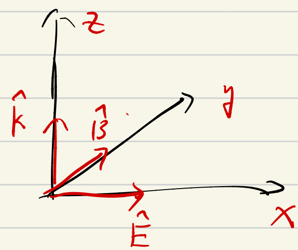


$$E = \hat{\varepsilon}\left(E e^{i(kx-\omega t)} + E^{*} e^{-i(kx-\omega t)}\right)$$

$$\mathcal{H} = \frac{1}{2}\int d^3r \left[\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right]$$ — 3D

$\hat{k} = \hat{z}$, $\hat{E} = \hat{x}$, $\hat{B} = \hat{y}$





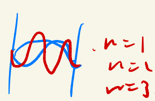
Cubic $L \times L \times L = V$

maxwell eq.

$$E(z,t) = \left(\frac{2\omega^2}{V\varepsilon_0}\right)^{\frac{1}{2}} q(t) \sin(kz)$$

$n \in \mathbb{Z}^+$

$k = \frac{\omega}{c}$ → single mode. $\omega \Leftrightarrow \omega_n = c\frac{n\pi}{L} = ck_n$



$$B(z,t) = \left(\frac{\mu_0 \varepsilon_0}{k}\right)\left(\frac{2\omega^2}{V\varepsilon_0}\right) \underbrace{\dot{q}(t)}_{} \cos(kz).$$

$p(t)$.

plug E. B. into $\mathcal{H}$.

$$\mathcal{H}_{EM} = \frac{1}{2}\left(\hat{p}^2 + \omega^2 q^2\right) \Leftrightarrow \mathcal{H}_{HO} = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 q^2$$

$$= \frac{1}{2m}\left[p^2 + m^2\omega^2 q^2\right]$$

$\Rightarrow$ The [illegible] condition.

$$[\hat{q}, \hat{p}] = i\hbar$$

define

$$\begin{cases} a = (2\hbar\omega)^{-\frac{1}{2}}(\omega\hat{q} + i\hat{p}) \\ a^+ = (2\hbar\omega)^{-\frac{1}{2}}(\omega\hat{q} - i\hat{p}) \end{cases}$$

$$\Rightarrow \begin{cases} \hat{E} = E_0 (a^+ + a) \sin(kz) \\ \hat{B} = \frac{1}{2} B_0 (a - a^+) \cos(kz) \end{cases}$$

ignore.

$$\frac{E_0}{c} = B_0$$

optical field

$\Leftarrow$

$$\hat{E} = \Omega(a^+ + a)$$

where

$$[a, a^+] = 1.$$

$\Rightarrow$ $\mathcal{H}_{EM} = \hbar\omega(a^+a + 1/2)$.

where $a^+a = \hat{n}$, number operator.

$\hat{n}|\Psi_n\rangle = n|\Psi_n\rangle$

T. independent

$E(t)$ $B(t)$
a, a^+. T. D.

Dynamics of a, a^+.

Heisenberg picture.

$$\frac{da}{dt} = \frac{i}{\hbar}[\hat{\mathcal{H}}_{EM}, a] = -i\omega a.$$

$$\Rightarrow \begin{cases} a(t) = a(0)\, e^{-i\omega t} \\ a^+(t) = a^+(0)\, e^{i\omega t} \end{cases}$$

$\hat{n} = a^+a = a^+(0)\,a(0)$, $\mathcal{H}_{EM}$. T. independent.

— Fock state.

Number state $|n\rangle$.

$\hat{\mathcal{H}}_{EM}|n\rangle = E_n|n\rangle$.

$$E_n = \hbar\omega\left(n+\tfrac{1}{2}\right)$$

$$\hbar\omega\left(a^\dagger a + \tfrac{1}{2}\right)|n\rangle = \hbar\omega\left(n+\tfrac{1}{2}\right)|n\rangle$$

$$\Rightarrow \quad a^\dagger a\,|n\rangle = n\,|n\rangle$$

$$\hat{n}\,|n\rangle = n\,|n\rangle$$

$a^\dagger \to n+1$. $\quad a \to n-1$.

$$\begin{cases} a\,|n\rangle = \sqrt{n}\,|n-1\rangle \\ a^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle \end{cases}$$

Ladder operator

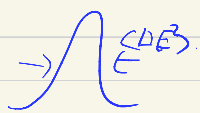
creation / annihilation operators.

— Quantum fluctuation

essence.

Quantum entanglement

$|0\rangle$: vacuum



E-field $\to$ $\acute{E} = \langle 0|\hat{E}|0\rangle \sim \boxed{\langle \Delta \hat{E}^2\rangle}$

$$\langle n|\hat{E}|n\rangle = E_0^2 \sin kz \left[\underbrace{\langle n|a|n\rangle}_{\substack{\langle n|n-1\rangle \\ =0}} + \underbrace{\langle n|a^\dagger|n\rangle}_{=0}\right]$$

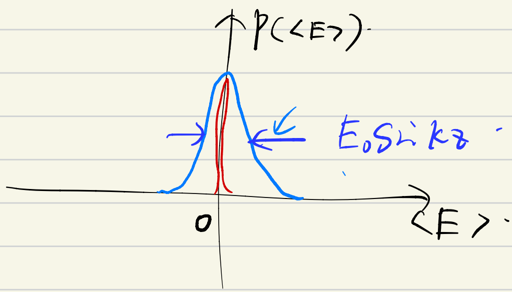
$$= 0$$

$$\langle \Delta E^2 \rangle = \langle E^2 \rangle - \underbrace{\langle E \rangle^2}_{=0}$$

$$= 2 E_0^2 \sin^2 kz \, (n + 1/2)$$

even $n = 0$ $\quad \langle \Delta E^2 \rangle \neq 0$

$$\langle \Delta E^2 \rangle_{n=0} = E_0^2 \sin^2 kz .$$



field fluctuation.

— Zero point energy.

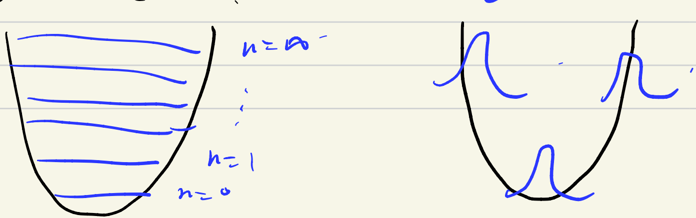
$E_n = \hbar\omega(n + 1/2)$: Einstein..

Time / space average of fluctuating energy

$$\boxed{\tfrac{1}{2}\hbar\omega}$$

— Coherent state.

Fock state $\qquad$ Coherent state



coherent state $|\alpha\rangle$

相干态. $\alpha \in \mathbb{C}$

$$|\alpha\rangle \equiv e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$

classical analogus. , laser.

Light - atom stimulated emission radiation.

- quasi-orthogonal.

$$\langle\alpha|\beta\rangle = e^{-\frac{1}{2}|\alpha|^2 - \frac{1}{2}|\beta|^2 + \alpha^*\beta} \neq 0$$

$$|\langle\alpha|\beta\rangle|^2 = e^{-|\alpha-\beta|^2}$$

If $|\alpha-\beta|^2 \gg 1$, $|\langle\alpha|\beta\rangle|^2 \to 0$.

- $a|\alpha\rangle = \alpha|\alpha\rangle$ ☆.

$a|0\rangle = 0$.

- Average photon #.

$$\bar{n} = \langle\alpha|\hat{n}|\alpha\rangle = \langle\alpha|a^\dagger a|\alpha\rangle = \alpha^*\alpha = |\alpha|^2$$

$$\Delta n = \langle\alpha| n^2 |\alpha\rangle - \langle\alpha|\hat{n}|\alpha\rangle^2$$

$$= \langle\alpha| a^\dagger a a^\dagger a |\alpha\rangle - |\alpha|^2$$

$a a^\dagger - a^\dagger a = 1. \Rightarrow a a^\dagger = 1 + a^\dagger a.$

$$= \langle\alpha| a^\dagger a + a^\dagger a^\dagger a a |\alpha\rangle. - |\alpha|^2$$

$$= \alpha^* \alpha^* \alpha \alpha = |\alpha|^4.$$

- $$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$

$$= \sum_{n=0}^{\infty} C_n |n\rangle \quad , \sum_{n=1}^{\infty} C_n^* C_n = 1.$$

$$P_n = C_n^* C_n = e^{-|\alpha|^2} \frac{(|\alpha|^2)^n}{n!}$$

$$= e^{-\bar{n}} \frac{(\bar{n})^n}{n!}$$

Poisson distribution.

$$\sqrt{\langle \Delta n^2 \rangle} = \bar{n} = |\alpha|^2$$

$$\langle \Delta n^2 \rangle = |\alpha|^4.$$

— Quantum ? classical ?

$|0\rangle$. $|\alpha\rangle$

Quantum. classical,

$|\alpha\rangle \rightarrow$ Laser. classical light

$\langle \Delta E^2 \rangle$ is smallest. $\langle \Delta B^2 \rangle = 1/4$.

$|n\rangle \rightarrow$ Quantum light, certain quantum effect

Laser Cooling.

Atoms gas.

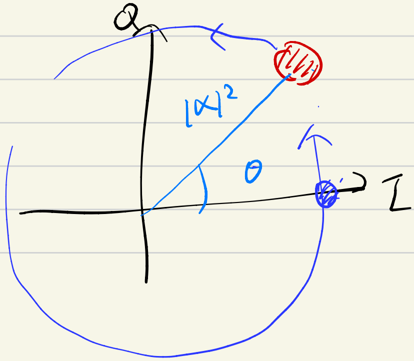
$|\alpha\rangle$.

3 mK.

199X N prize.

BEC. 1999 Wolfgang, Eric Cornell.

H_2. MIT JILA. Rice

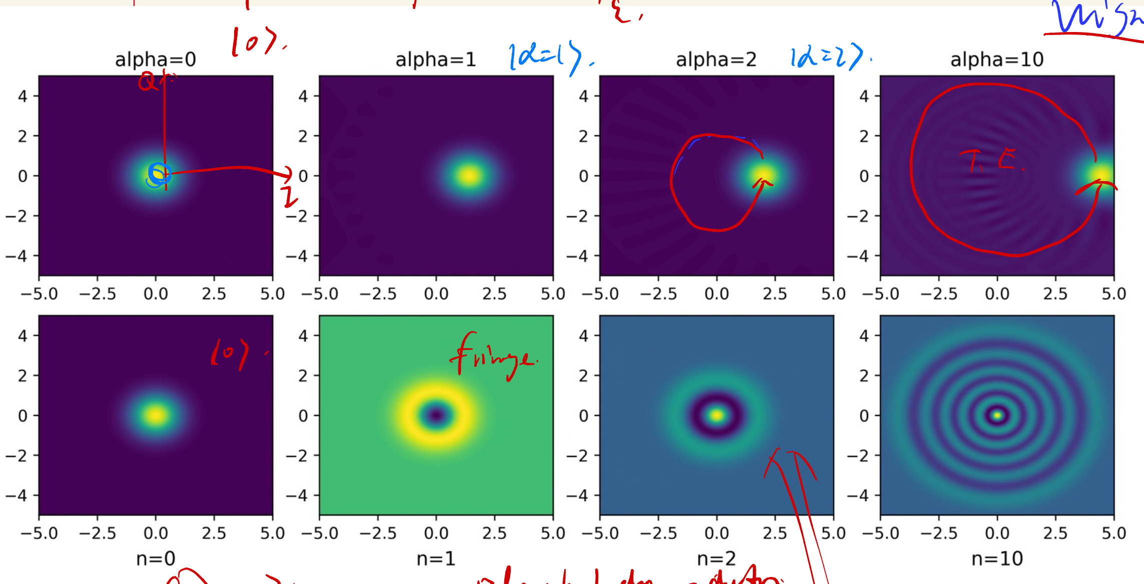


$\alpha = |\alpha| e^{i\theta}$.

phase space. $p \uparrow \rightarrow q$. {quasi-prob. distr.}

Wigner

$|0\rangle$. $|\alpha=1\rangle$. $|\alpha=2\rangle$.



T.E.

$|0\rangle$.

fringe.

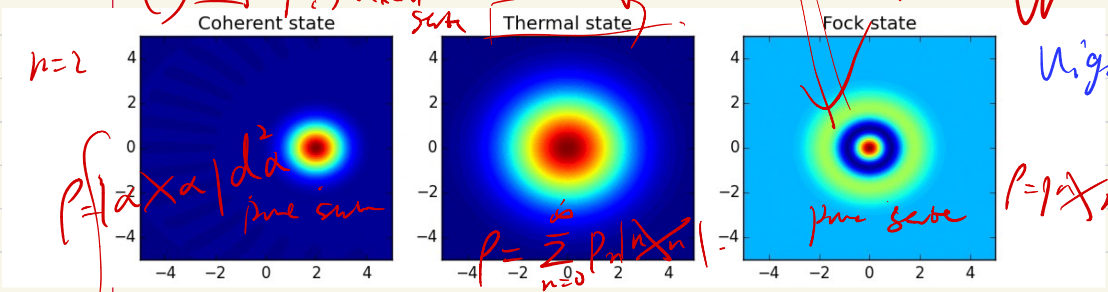
ρ: mixed state

Black body radiation

n=2

W

Wigner



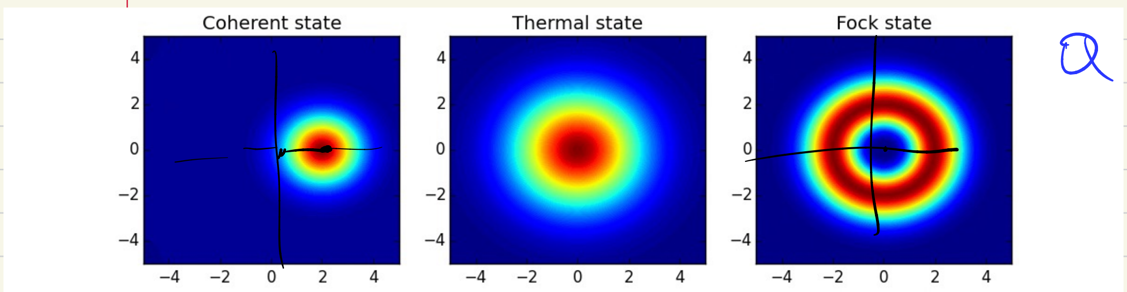
$\rho = \int |\alpha\rangle\langle\alpha|\, d\alpha^2$ pure state

$\rho = \sum_{n=0}^{\infty} P_n |n\rangle\langle n|$

pure state

$\rho = |n\rangle\langle n|$

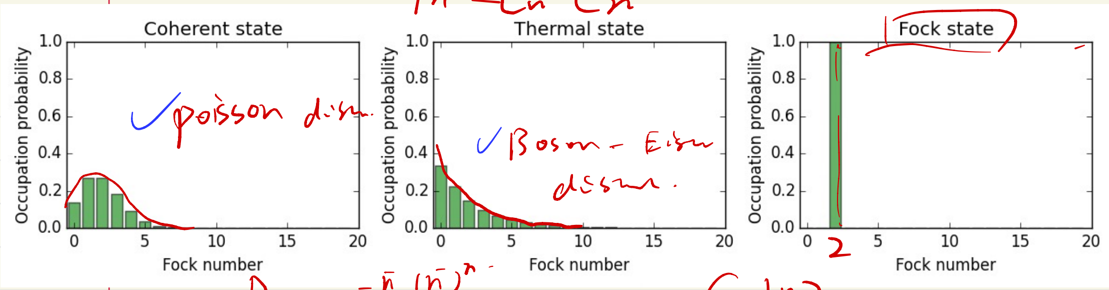
Q



$Q(\beta) = \langle\beta|\psi\rangle\langle\psi|\beta\rangle$.

Thermal state: Chaotic state

$P_n = C_n^* C_n$



$P_n = e^{-\bar{n}} \frac{(\bar{n})^n}{n!}$

$C_n|n\rangle$.